TEAM-NET "Near-term quantum computers: challenges, optimal implementations and applications" Quantum Error Correction Group

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I. MOTIVATION AND GENERAL GOAL

Quantum technology promises truly exciting opportunities for computing, metrology, cryptography, the simulation of materials, and optimization. At the same time, tools from quantum information theory are seeking tasks outside of their original field of application, in particular in mathematics and solid state physics.

The research of this group will chiefly focus on the interplay between quantum error correction with entanglement, many-body systems, mathematical physics, and non-commutative algebra. The primary area of reseach is on the following topics:

Practical quantum codes: The group will develop experimentally feasible implementations of quantum codes in higher spin systems and of holographic codes in ultracold atoms. On the theoretical side, the group will investigate the nonlocal properties of quantum codes, in settings where subsets of parties can perform joint measurements. Finally, a long-term goal is to establish a systematic approach to semidefinite programming bounds on the distance of codes.

Entanglement detection and geometry of quantum states: Building on recent work of the group leader [1], we will study the task of non-linear entanglement detection with many copies of a state and will provide a general characterization of the entanglement in Werner states. We will also study the robustness properties of reconstruction methods for ground states of almost frustration-free Hamiltonians.

Mathematical physics and non-commutative optimization: We will study tensor polynomial identities and Haar integration for the commutant of the walled Brauer Algebra. Furthermore, a semidefinite programming hierarchy for the spectral quantum marginal problem with overlapping reduced density matrices will be developed. Lastly, the group aims at developing algorithms for computationally hard matrix functions on near-term quantum computers.

The above topics are both timely and crucial: it ensures that innovative concepts from quantum error-correction and quantum information find their abstraction in mathematics, while pushing the frontiers towards the realization of quantum error correction and near-term quantum computing, as well as the understanding of quantum many-body systems. This proposed research aims to lead the way, and linking up with the research groups of TEAM-NET "Near-term quantum computers: challenges, optimal implementations and applications" will prove to be a game changer.

II. RESEARCH OBJECTIVES

A. Objective 1: Practical quantum codes

We begin by understanding higher spin quantum codes of the Gottesman-Kitaev-Preskill type [2], as part of a strategy to develop universal quantum computation with ultracold atoms. Then we transition to find experimentall friendly implementation of the holographic code in the laboratory. Here making use of the relation of stabilizer states to graph states will be of advantage [3]. For practical purposes it will be important to find the simplest experimental realization of such a graph state and we aim at an implementation targeted at ultracold atoms that could eventually be implemented by the research group of Prof. Fred Jendrzejewski at the University of Heidelberg. This would yield the first realization of a holographic code in a lab and could provide a potential tool to study information processes in black holes [4]. We collaborate with Dr. Valentin Kasper from ICFO Barcelona.

Specific tasks:

1. Realize the tensor network contraction as a graph state, identify the experimentally most friendly graph through local complementation.
2. Demonstrate the holographic properties through explicit recovery gates.
3. Formulate an experimental cook book for an implementation with ultracold atoms.

B. Objective 2: Nonlocality in quantum codes

Building on recent work by the group leader [5], we aim to characterize what other quantum features quantum error correcting codes exhibit: here we investigate the non-local properties of quantum codes and multipartite quantum states, where subsystems can act jointly to perform measurements, a hitherto entirely unexplored scenario. We will aim to integrate matrix polynomial identities [6] into the Navascues-Pironio-Acín hierarchy [7], taking advantage of a type of frustration in Bell non-locality. We collaborate with Dr. Alex Pozas from Universidad Complutense Madrid.

Specific tasks:

1. Identify simplest code where overlapping measurements can be applied.
2. Incorporate covariance matrix sampling or polynomial identities.
3. Determine symmetries for problem reduction and implementation with sparse semidefinite programming.

C. Objective 3: Non-linear entanglement detection

We will develop methods for non-linear entanglement detection. Building on recent work by the group leader [1], the aim is to characterize how to detect entanglement with a restricted set of witnesses, namely trough non-linear trace polynomials. This then provides a method by which *multi*partite witnesses can be used for *bi*partite entanglement detection. This will allow to detect entanglement more efficiently through the use of many copies of a quantum state and will provide general mathematical tools that will be useful in quantum information theory. This will likely also yield building blocks to find higher order bounds on the distance of quantum codes, a method that is developed for classical codes [8] but not yet for quantum codes, and is closely related to entanglement monogamy relations [9]. We collaborate with the group of Prof. Karol Życzkowski.

Specific tasks:

1. Identify the key feature(s) needed for non-linear witnesses of trace polynomial type.
2. Characterize its detection power, find completeness results or provide counterexamples.
3. Understand whether this approach can be generalized, e.g. with Clifford-invariant witnesses, and identify further applications in quantum error correction and monogamy of entanglement.

D. Objective 4: Mathematical physics and non-commutative optimization

We will study methods to understand the entanglement present in Werner states. This class of states can be characterized by only $n!$ parameters where n is the number of systems [10]. It is natural to expect that their entanglement properties can be described independently of the the local Hilbert space dimension. Making use of a Gram matrix representation [11], the task of finding a Werner state witness can likely be formulated in terms of a commutative or non-commutative optimization problem [12]. Here it is probable that the tools of semidefinite programming hierarchies [13] can find an application. We collaborate with Dr. Jurij Volcic from the University of Copenhagen, Prof. Igor Klep from the University of Ljubljana, and Dr. Victor Magron from LAAS-CNRS & Institute of Mathematics from Toulouse.

Specific tasks:

1. Understand how Werner state entanglement can be characterized in a dimension-free manner.
2. Map state separability to positive functionals on a Gram matrix.
3. Formulate the task of finding an entanglement witness to a optimization problem.

III. ORGANIZATION OF WORK AND COLLABORATORS

The Quantum Error Correction Group will consist of 6 people: one project leader, two supporting scientists (postdoctoral researchers), and three PhD students. The group will operate as part of the TEAM-NET collaboration, whose expected duration is until October 2023. Each PhD student will focus on one research objective, with postdoctoral students in supporting roles and additional external advisors.

To realize our aims, a strong network of collaborators is key: first of all the integration into the TEAM-NET program. Especially working with the group of Prof. Karol Życzkowski at the Jagiellonian University in Krakow, and with the group of Dr. Michał Oszmaniec in Warsaw will likely be benefitial due to the overlap in research areas.

Regarding international collaborations, the group currently works with Simeon Ball, Valentin Kasper, and Andreas Winter (all from Barcelona), Markus Grassl and Michał Studziński (Gdansk), Otfried Gühne and Chau Nguyen (Siegen), Victor Magron (Toulouse), Igor Klep (Ljubljana), Nikolai Wyderka (Düsseldorf), Claudio Procesi (Rome), Jurij Volčič (Copenhagen), Alex Pozas and Angelo Lucia (Madrid), as well as Niklas Johansson and Jan-Åke Larsson (Linköping).

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