Quantum Advantage in Thermodynamics

Quantum Resources Group

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Near-Term Quantum Computing Workshop

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Personal Background

Education & Work

Studies in Mathematics & Physics at TU Munich (TUM)
International Academic Stays in the Americas (US & Brazil)
Thesis on Symmetries in Open Quantum Systems at TUM and MPQ
Visiting Researcher at Center for Quantum Technologies (CQT) in Singapore
Contract with Quantum Resources Group at Jagiellonian University in Cracow

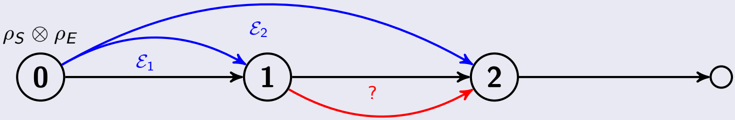
Interests

Mathematical Physics, Quantum Information, Open Quantum Systems, Decoherence, Quantum Thermodynamics, Resource Theories, Quantum Control

Projects

Product of Probability Vectors and Quantum States with Karol Życzkowski
Quantum Advantage in Simulation of Stochastic Processes with Kamil Korzekwa

Quantum Dynamical Semigroups



Markovian Postulate: Semigroup Structure of Family $\mathcal{E}_t$

$$\mathcal{E}_0 = I \quad \mathcal{E}_t \circ \mathcal{E}_s = \mathcal{E}_{s+t} \quad s, t \in \mathbb{R}_+$$

Topology: Continuity $\Rightarrow$ Differentiability & Existence of Generator $\mathcal{L} := \partial_t \mathcal{E}_{t=0}$

$$\mathcal{E}_t = \exp(t\mathcal{L}) = \sum_{k \in \mathbb{N}_0} t^k \mathcal{L}^k / k!$$

Dynamics in Continuous Time via *Quantum Master Equation* $\dot{\rho} = \mathcal{L}(\rho)$

NP-Hardness of Markovianity Problem: Question of Embeddability

Lindbladian Superoperator

Derivation via Differential Quotient according to Definition $\mathcal{L} := \partial_t \mathcal{E}_{t=0}$

$$\mathcal{L}(\rho) = \mathcal{F}(\rho) - (G\rho + \rho G^\dagger) \quad \text{with} \quad \mathcal{F} \text{ CP}, G \in \boldsymbol{\mathcal{L}}(\boldsymbol{H})$$

$\mathcal{E}$ TP $\Rightarrow$ IP Property $\mathcal{E}^\ddagger(I) = \exp(t\mathcal{L}^\ddagger)(I) = I \Rightarrow \mathcal{L}^\ddagger(I) = 0 \Rightarrow \mathcal{F}^\ddagger(I) = G + G^\dagger$
Splitting $G = (G + G^\dagger)/2 + (G - G^\dagger)/2 \Rightarrow G = \mathcal{F}^\ddagger(I)/2 + iH$ for $H = H^\dagger$ (wlog)
Choosing Kraus Representation for *Quantum Transition Map* $\mathcal{F}(\rho) = \sum_k F_k \rho F_k^\dagger$

$$\boxed{\mathcal{L}(\rho) = -i[H, \rho] + \sum_k F_k \rho F_k^\dagger - \{F_k^\dagger F_k, \rho\}/2}$$

Gorini-Kossakowski-Sudarshan-Lindblad Semigroup Generator

H *Effective Hamiltonian* governing Coherent Part of Evolution
F_k *Lindblad (Noise) Operators* responsible for Incoherent Part
Separation $\mathcal{L}(\rho) = \mathcal{H}(\rho) + \mathcal{D}(\rho)$ into $\mathcal{H}(\rho) := i[\rho, H]$ and *Dissipator* $\mathcal{D}(\rho)$

Dynamical Symmetries

Dynamical Symmetries in Closed and Open Systems

Dynamical Symmetry Groups $\boldsymbol{G}$:
Invariance of Time Evolution Generator under Group Action

Closed Case

$\forall\, U \in \boldsymbol{G} : \; U^\dagger H U = H$ or equivalently $[H, U] = 0 = [H, G]$
Symmetry Generators $iG \in \mathfrak{g} = \mathrm{Lie}(\boldsymbol{G}), U(\phi) = \exp(i\phi G)$
Elements of Commutant $H' := \{O \in \boldsymbol{\mathcal{L}}(\boldsymbol{H}) |\; [H, O] = 0\}$

Open Case

$\forall \mathcal{U} \in \boldsymbol{\mathcal{G}} : \mathcal{U}^\ddagger \mathcal{L} \mathcal{U} = \mathcal{L}$ or equivalently $[\mathcal{L}, \mathcal{U}] = 0 = [\mathcal{L}, \mathcal{G}]$
Symmetry Generators $\mathcal{G} \in \mathrm{Lie}(\boldsymbol{\mathcal{G}}), \mathcal{U}(\phi) = \exp(\phi \mathcal{G})$
Elements of Commutant $\mathcal{L}' := \{\mathcal{O} \in \boldsymbol{\mathcal{L}}(\boldsymbol{\mathcal{L}}(\boldsymbol{H})) |\; [\mathcal{L}, \mathcal{O}] = 0\}$

Commutants form Lie Algebras due to Jacobi Identity:

$$\forall \mathcal{O}_1, \mathcal{O}_2 \in \mathcal{L}' : [[\mathcal{O}_1, \mathcal{O}_2], \mathcal{L}] = [\mathcal{O}_1, [\mathcal{O}_2, \mathcal{L}]] - [\mathcal{O}_2, [\mathcal{O}_1, \mathcal{L}]] = 0$$

Relation between Symmetry and Conservation

Quantum Version of Noether Theorem

Closed Case

$$U^\dagger H U = H \Leftrightarrow [H, G] = 0 \Leftrightarrow \mathcal{H}^\ddagger(G) = \dot{G} = 0$$

Open Case

$$\mathcal{U}^\ddagger \mathcal{L} \mathcal{U} = \mathcal{L} \Leftarrow [H, G] = 0 = [F, G] = [F^\dagger, G] \; \forall F \Rightarrow \mathcal{L}^\ddagger(G) = \dot{G} = 0$$

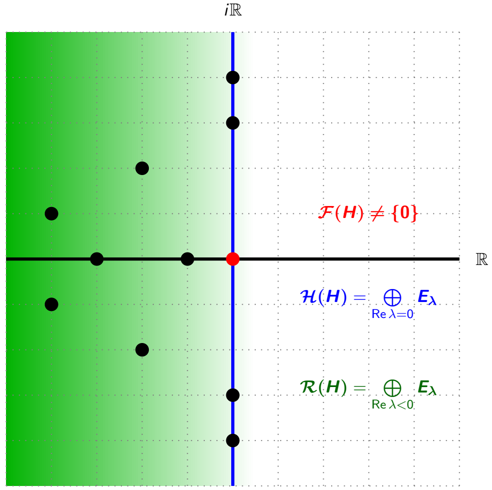
Special Case: Lower-Level Unitaries

$$\mathcal{U}(\cdot) = \mathrm{Ad}_U(\cdot) = U(\cdot)U^\dagger \Rightarrow \boxed{\mathcal{G}(\cdot) = \mathrm{ad}_{iG}(\cdot) = i[G, \cdot]} \text{ due to}$$

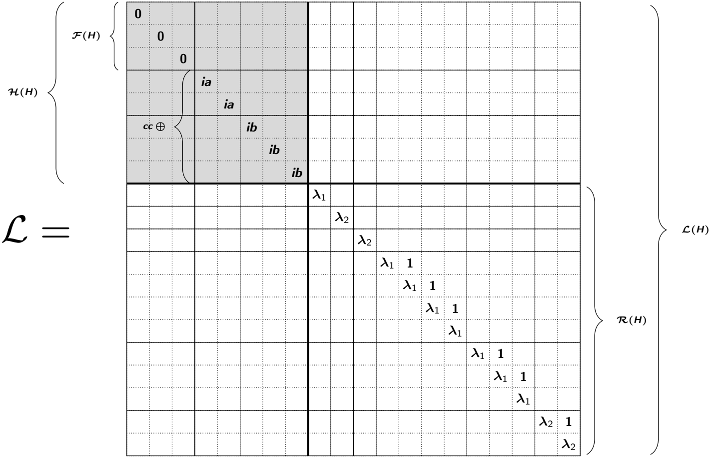
$$e^{+i\phi G}(\cdot)e^{-i\phi G} = \mathrm{Ad}_{e^{i\phi G}}(\cdot) = e^{\mathrm{ad}_{i\phi G}}(\cdot)$$

Quest for New Symmetries: Structure of Commutant $\{\mathcal{O} \in \boldsymbol{\mathcal{L}}(\boldsymbol{\mathcal{L}}(\boldsymbol{H}))| \; [\mathcal{L}, \mathcal{O}] = 0\}$

Spectrum of Lindbladian

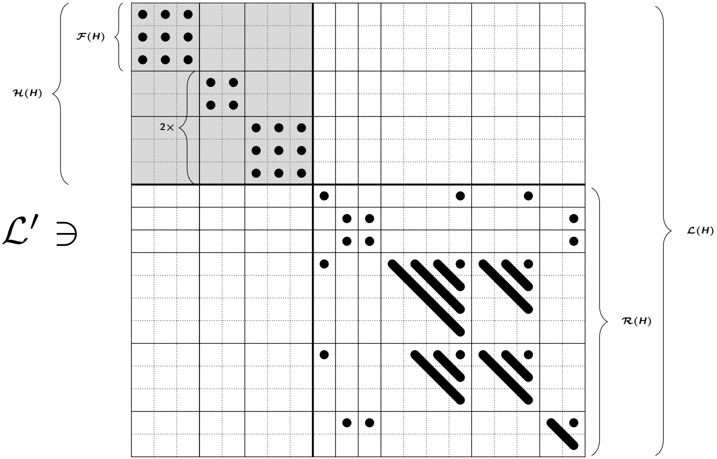


Normal Form of Lindbladian



Commutant Structure

$\mathcal{L}' \ni$



Quantum Advantage

Quantum Embeddability of Stochastic Matrix P: Simulation by Markovian Channel $\mathcal{E}$

Definition: $P_{i|j} = \langle i|\mathcal{E}(|j\rangle\langle j|)|i\rangle$ with $\mathcal{E}$ Member of Quantum Dynamical Semigroup
Time-Homogeneous Case considered at Beginning: $P_{i|j} = \langle i|\exp\mathcal{L}(|j\rangle\langle j|)|i\rangle$

Open Question: Properties of P for Quantum Embeddability

Conditions known for Classical Embeddability
(Existence of Markov Generator Q: $P = \exp Q$)
Search for Coherification $\mathcal{E}$ (Markovian Channel) for given P
via Majorization Bounds on Spectrum of Jamilkowski State

Quantum Thermodynamics: Memory Advantage in State Transformations

Maximal Quantum Advantage for Uniform Fixpoints: Memoryless Quantum Channels can simulate *all* Classical Stochastic Processes
Result by Numerics: Maximal Quantum Advantage for Qubits and given Fixpoint
Goal: Identification of physically meaningful time-dependent Lindbladians for Markovian Cooling to "other Side" of Gibbs State